Finite Formulation of Surface Impedance Boundary Condition

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In several electromagnetic applications field quantities are confined in layers which are thin with respect to other geometrical dimensions. The numerical solution of these phenomena has led to the development of special formulations. Among these, the surface impedance boundary conditions has been extensively developed in the past decades, often coupling it to other techniques for the analysis of volumes like finite elements or boundary elements method. In this paper a finite formulation of the surface impedance boundary condition is presented and its application to the analysis of induction heating problems is proposed. The novelty of the paper lies in the definition of the operative quantities present in the formulation and in its development toward an iterative technique which takes into account magnetic nonlinearity.

***Index Terms*—Impedance Boundary Condition, Boundary Element Method, Hybrid Techniques, Induction Heating**

I. INTRODUCTION

THE surface impedance boundary condition (SIBC) is a technique which can be used to treat electromagnetic phenomena in time-varying electromagnetics. Some very important papers have discussed its main aspects both from the theoretical and from the implementation viewpoint, like in [1], [2], [3]. The SIBC formulation can treat both thin layers of eddy currents but can also be applied to the case of truly thin conductors like in the case of magnetic and conductive shields as shown in [4].

Induction heating is an application where the most important phenomena are confined in a very thin surface layer. This is mainly due to the frequency values used, in the kilohertz range, and because most often workpieces are ferromagnetic. In the past, part of the Authors of the present paper have developed a particular formulation to take into account this peculiarity by means of a one dimensional solution coupled to a three-dimensional finite element analysis of the whole structure [5]. A first harmonic sinusoidal approach is used to approximate nonlinear effects [6], [7]. The present work would make a synthesis of this previous experience by introducing the use of SIBC in the simulation of skin effect but keeping, at least in an approximate way, the effect of magnetic nonlinearity.

II. BEM-SIBC HYBRID FORMULATION

The considered domain is made by a magnetic and conductive volume surrounded by a region where source conductors with impressed currents are present. It is considered that the external region is studied by means of a BEM technique formulated in terms of reduced magnetic scalar potential. It is also assumed that the volume is replaced by its boundary.

A. BEM formulation

By considering that the magnetic field is decomposed in two parts $\vec{H} = -\nabla\psi + \vec{H}_S$ where $\vec{H}_S$ is the magnetic field created by coils with imposed current sources, the reduced magnetic scalar potential ψ can be computed by BEM equation on the external surface of the meshed domain by assuming that:

- ψ is evaluated in the center point of faces;
- the term $\frac{\partial\psi}{\partial n}$ is uniform on each face,

the BEM equation [8], can be written in matrix form as:

$$\mathbf{H}\boldsymbol{\psi} + \mathbf{W}\frac{\partial\boldsymbol{\psi}}{\partial\mathbf{n}} = 0 \tag{1}$$

where:

$$(\mathbf{H})_{ij} = \delta_{ij}\alpha - \int_{S_j} \frac{\partial G(\vec{r}, \vec{r}')}{\partial n}\mathrm{d}S$$

$$(\mathbf{W})_{ij} = \int_{S_j} G(\vec{r}, \vec{r}')\mathrm{d}S$$

where α is the portion of solid angle seen by the point $\vec{r}$, G is the Green function, δ_{ij} is the Kronecker function, $\mathbf{H}$ and $\mathbf{W}$ are square matrices whose dimensions are $(N_F \times N_F)$ being N_F the number of faces on the surface.

B. SIBC formulation

Sinusoidal varying quantities are considered so that complex form can be used. Under these assumptions, phasor quantities decay exponentially inside the material. The shell is discretised by means of two surface grids connected by duality relationship: configuration variables are associated to primal grid and source variables to the dual one [9]. Even if these cells are represented on the two-dimensional objects, it is considered that each edge is the trace of a *shell face* Σ extending along the depth up to the level where all fields are null. The global variables associated to the geometrical dimensions are reported in Table I. Due to the geometrical and field peculiarities, the electromagnetic induction law on each of the faces on the shell is given by:

$$\mathbf{C}e = -\mathrm{j}\omega\phi \tag{2}$$

where $\mathbf{C}$ is a $(N_F \times N_E)$ matrix containing the incidence of primal edges on faces, being N_E the number of edges on the surface and ϕ is the phasor of the magnetic flux incident on the face. The Ampère's law is applied on dual faces. By considering the circulation equation on shell faces Σ, it can be seen how this degenerates because only one of the edges

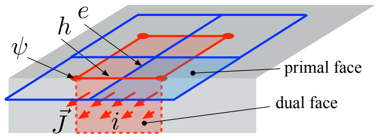

Fig. 1: Problem discretisation, primal grid in blue and dual grid in red. Variables: ψ on dual nodes, h on dual edges, $\vec{J}$ and i on dual faces, e on primal edges.

TABLE I: Global variables and their associated domains

Global Variable	Unit	definition	grid	element
electro-motive force	V	$e = \int_L \vec{E} \cdot \mathrm{d}\vec{L}$	primal	2D edge
magnetic shell flux	Wb	$\varphi = \int_\Sigma \vec{B} \cdot \mathrm{d}\vec{S}$	primal	shell face
incident magnetic flux	Wb	$\phi = \int_S \vec{B} \cdot \mathrm{d}\mathrm{d}\vec{S}$	primal	2D face
magnetic scalar pot.	A	ψ	dual	2D node
magneto-motive force	A	$h = \int_{\tilde{L}} \vec{H} \cdot \mathrm{d}\vec{L}$	dual	2D edge
current	A	$i = \int_{\tilde{\Sigma}} \vec{J} \cdot \mathrm{d}\vec{S}$	dual	shell face

is active. According to this assumption the electromagnetic induction law becomes $i = h$. The behaviour of the field quantities inside the shell can be obtained by their analytical solutions under the assumption of quantities varying as:

$$J(y) = J_0 \mathrm{e}^{-\frac{(1+\mathrm{j})}{\delta} y} = J_0 \mathrm{e}^{-\frac{y}{\delta}} \mathrm{e}^{-\mathrm{j}\frac{y}{\delta}} \tag{3}$$

where J_0 is the current density on the surface of the material. The electric field on the surface can be obtained by e on a primal edge as $E_0 = \frac{e}{\lambda}$ where λ is the length of the primal edge. By exploiting the constitutive relation $J_0 = \sigma E_0$ the constitutive relation between e and the corresponding i along the dual face twined with the edge can be written as:

$$i = \int_{\tilde{L}} \int_0^{+\infty} J_0 \mathrm{e}^{-\frac{y}{\delta}} \mathrm{e}^{-\mathrm{j}\frac{y}{\delta}} \mathrm{d}y \mathrm{d}L = \tilde{L} \frac{\delta}{1+\mathrm{j}} J_0 = \frac{\sigma \tilde{L} \delta}{L(1+\mathrm{j})} e \tag{4}$$

where $\tilde{L}$ is the breadth of the dual face. Equation (4) defines the admittance related to each couple primal edge/dual face on the shell. The relation can be inverted in order to obtain the diagonal matrix $\mathbf{Z}$.

C. Hybrid solution

The topological and constitutive equations on the shell are assembled to solve the current flow problem using a technique similar to the mesh analysis:

$$\mathbf{CZC}^T \mathbf{i}_{\text{mesh}} = -\mathrm{j}\omega\boldsymbol{\phi} \tag{5}$$

where $\mathbf{i}_{\text{mesh}}$ is the array of fictitious currents associated to each primal face. The coupling terms between the circuit equation (5) and the BEM one (1) are the incident magnetic flux that is related to the normal derivative of the reduced magnetic scalar potential as:

$$\boldsymbol{\phi} = \mu_0 \mathbf{A} \left(-\frac{\partial \psi}{\partial \mathbf{n}} + \mathbf{H}_{Sn} \right) \tag{6}$$

where $\mathbf{A}$ is the array of primal face areas, and by the Ampère's law applied on dual edges:

$$\mathbf{i} = -\tilde{\mathbf{G}}\boldsymbol{\psi} + \mathbf{h}_S \Rightarrow \mathbf{i} = -\mathbf{C}^T \boldsymbol{\psi} + \mathbf{h}_S \tag{7}$$

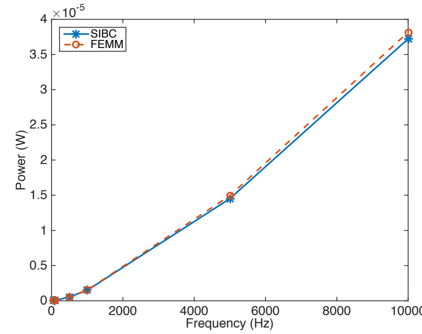

Fig. 2: Comparison of the power loss calculated using a 2d axysimmetric code and BEM-IBC hybrid formulation.

where subscript S refer to the fields created by imposed current source. The final assembled version of the equations become:

$$\begin{bmatrix} \mathbf{H} & \mathbf{W} \\ -\mathbf{CZC}^T & \mathrm{j}\omega\mu_0\mathbf{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi} \\ \frac{\partial \boldsymbol{\psi}}{\partial \mathbf{n}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ -\mathrm{j}\omega\mu_0\mathbf{AH}_{Sn} - \mathbf{CZh}_S \end{bmatrix}$$

The previous system of equations can be solved directly or iteratively through the Schur complement technique [11].

The proposed technique has been applied to the study of a ferromagnetic slab excited by a circular coil.The solution has been compared with a 2D axisymmetric FEM one at different frequency values ranging from 50 Hz to 10 kHz obtaining a very good agreement on field and integral quantities (Fig. 2). The extension of the proposed approach to nonlinear case and to unstructured surface discretizations will be presented at the Conference.

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